Number Bases

- Number bases are number systems consisting of digits from 0 to 9.
- The number systems are made up of numbers with various bases.
- Base ten is a decimal number system used widely in daily life.

Number base	Digit
Base 2	0, 1
Base 3	0, 1, 2
Base 4	0, 1, 2, 3
Base 5	0, 1, 2, 3, 4
Base 6	0, 1, 2, 3, 4, 5
Base 7	0, 1, 2, 3, 4, 5, 6
Base 8	0, 1, 2, 3, 4, 5, 6, 7
Base 9	0, 1, 2, 3, 4, 5, 6, 7, 8
Base 10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9

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Examples of numbers that represent numbers in base two up to base ten.

Number base	Number	
2	10_2	1001_2
3	21_3	1201_3
4	23_4	213_4
5	41_5	342_5
6	35_6	4510_6
7	64_7	463_7
8	17_8	472_8
9	78_9	385_9
10	69_{10}	2893_{10}

Place values involved in numbers in base two up to base ten

Number base	a^n	Place value							
		a^7	a^6	a^5	a^4	a^3	a^2	a^1	a^0
Base 2	2^n	128	64	32	16	8	4	2	1
Base 3	3^n	2187	729	243	81	27	9	3	1
Base 4	4^n	16384	4096	1024	256	64	16	4	1
Base 5	5^n	78125	15625	3125	625	125	25	5	1
Base 6	6^n	279936	46656	7776	1296	216	36	6	1
Base 7	7^n	823543	117649	16807	2401	343	49	7	1
Base 8	8^n	2097152	262144	32768	4096	512	64	8	1
Base 9	9^n	4782969	531441	59049	6561	729	81	9	1
Base 10	10^n	10000000	1000000	100000	10000	1000	100	10	1

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THE VALUE OF A PARTICULAR DIGIT IN A NUMBER IN VARIOUS BASES

❖ The value of a particular digit in a number is the multiplication of a digit and the place value that represents the digit.

Multiplication of digit and place value					Use of place value block				
$10\underline{1}0_2$	Number	1	0	<u>1</u>	0	$10\underline{1}0_2$	Number	1	0	<u>1</u>	0
	Place value	2^3	2^2	2^1	2^0		Place value	2^3	2^2	2^1	2^0
	Digit value			$1 \times 2^1 = 2$			Digit value			2	
$\underline{2}012_3$	Number	<u>2</u>	0	1	2	$\underline{2}012_3$	Number	<u>2</u>	0	1	2
	Place value	3^3	3^2	3^1	3^0		Place value	3^3	3^2	3^1	3^0
	Digit value	$2 \times 3^3 = 54$					Digit value	54			
$\underline{4}432_5$	Number	<u>4</u>	4	3	2	$\underline{4}432_5$	Number	<u>4</u>	4	3	2
	Place value	5^3	5^2	5^1	5^0		Place value	5^3	5^2	5^1	5^0
	Digit value	$4 \times 5^3 = 500$					Digit value	500			

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CONVERT A NUMBER IN BASE TEN TO ANOTHER BASE

Rajang River which is the longest river in Malaysia is 563 kilometres.
Convert 563_{10} to a number in

(a) base five. (b) base eight.

Solution:

(a) Base five

Division using place value

Place value	625	125	25	5	1
Step	The value of 625 is greater than 563	4 125) 563 − 500 63	2 25) 63 − 50 13	2 5) 13 − 10 3	3 1) 3 − 3 0
Base 5	0	4	2	2	3
Answer	4223_5				

563 is divided by the place value of 125. Its remainder is transferred to the previous place value for the next division until a zero remainder is obtained.

Alternative Method

Division using base value

5	563	
5	112	− 3
5	22	− 2
5	4	− 2
	0	− 4

Remainder

The digits are read from the bottom upwards.

The division is continued until digit zero is obtained.

$563_{10} = 4223_5$

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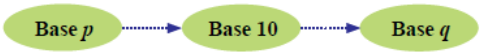
CONVERT A NUMBER IN A CERTAIN BASE TO BASE TEN AND THEN TO ANOTHER BASE

A number in base p can be converted to base ten and then to base q. In the process of converting a number in base two to base nine, the number in base two is converted to base ten and then to base nine.



Example 6

Convert 253_6 to a number in base nine.

Solution:

Step 1

Convert the number in base six to base ten.

Place value	6^2	6^1	6^0
Number in base 6	2	5	3
Value of number in base 10	$(2 \times 6^2) + (5 \times 6^1) + (3 \times 6^0)$ $= 105_{10}$		

Step 2

Convert the number in base ten to base nine.

$$\begin{array}{r|l} 9 & 105 \\ 9 & 11 \quad -6 \\ 9 & 1 \quad -2 \\ & 0 \quad -1 \end{array}$$

$253_6 = 105_{10} = 126_9$

Convert 334_5 to a number in base two.

Solution:

Step 1

Convert the number in base five to base ten.

Place value	5^2	5^1	5^0
Number in base 5	3	3	4
Value of number in base 10	$(3 \times 5^2) + (3 \times 5^1) + (4 \times 5^0)$ $= 94_{10}$		

Step 2

Convert the number in base ten to base two.

$$\begin{array}{r|l} 2 & 94 \\ 2 & 47 \quad -0 \\ 2 & 23 \quad -1 \\ 2 & 11 \quad -1 \\ 2 & 5 \quad -1 \\ 2 & 2 \quad -1 \\ 2 & 1 \quad -0 \\ & 0 \quad -1 \end{array}$$

$334_5 = 94_{10} = 1011110_2$

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ADDITION OF NUMBERS IN VARIOUS BASES

Vertical Form

(a) $110_2 + 111_2$

TIPS: Perform addition as usual and convert the values in base ten to base two.

$$\begin{array}{r} {\scriptstyle 1} \\ 1\,1\,0_2 \\ +\,1\,1\,1_2 \\ \hline 1\,1\,0\,1_2 \end{array}$$

$0 + 1 = 1_{10} = 1_2$

$1 + 1 = 2_{10} = 10_2$ ← Write 0 in the answer space. 1 is carried forward to the next place value.

$1 + 1 + 1 = 3_{10} = 11_2$

$110_2 + 111_2 = 1101_2$

(b) $673_8 + 175_8$

TIPS: Perform addition as usual and convert the values in base ten to base eight.

$$\begin{array}{r} {\scriptstyle 1\ 1} \\ 6\,7\,3_8 \\ +\,1\,7\,5_8 \\ \hline 1\,0\,7\,0_8 \end{array}$$

$3 + 5 = 8_{10} = 10_8$ ← Write 0 in the answer space. 1 is carried forward to the next place value.

$1 + 7 + 7 = 15_{10} = 17_8$

$1 + 6 + 1 = 8_{10} = 10_8$ ← Write 7 in the answer space. 1 is carried forward to the next place value.

$673_8 + 175_8 = 1070_8$

(c) $1837_9 + 765_9$

TIPS: Perform addition as usual and convert the values in base ten to base nine.

$$\begin{array}{r} {\scriptstyle 1\ 1\ 1} \\ 1\,8\,3\,7_9 \\ +\ \ 7\,6\,5_9 \\ \hline 2\,7\,1\,3_9 \end{array}$$

$7 + 5 = 12_{10} = 13_9$ ← Write 3 in the answer space. 1 is carried forward to the next place value.

$1 + 3 + 6 = 10_{10} = 11_9$ ← Write 1 in the answer space. The number 1 on the left is carried forward to the next place value.

$1 + 8 + 7 = 16_{10} = 17_9$

$1 + 1 = 2_{10} = 2_9$ ← Write 7 in the answer space. 1 is carried forward to the next place value.

$1837_9 + 765_9 = 2713_9$

Conversion of Base

$110_2 \rightarrow 6_{10}$

$111_2 \rightarrow +\,7_{10}$

13_{10}

2	13	
2	6	−1
2	3	−0
2	1	−1
	0	−1

$110_2 + 111_2 = 1101_2$

$673_8 \rightarrow 443_{10}$

$175_8 \rightarrow +\,125_{10}$

568_{10}

8	568	
8	71	−0
8	8	−7
8	1	−0
	0	−1

$673_8 + 175_8 = 1070_8$

$1837_9 \rightarrow 1411_{10}$

$765_9 \rightarrow +\,626_{10}$

2037_{10}

9	2037	
9	226	−3
9	25	−1
9	2	−7
	0	−2

$1837_9 + 765_9 = 2713_9$

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SUBTRACTION OF NUMBERS IN VARIOUS BASES

(b) $6241_7 - 613_7$

$$\begin{array}{r} {\scriptstyle 5\ 7\ 3\ 7} \\ 6\not{2}\not{4}1_7 \\ -\ \ 6\ 1\ 3_7 \\ \hline 5\ 3\ 2\ 5_7 \end{array}$$

Transfer numbers according to base seven. Use 7 to perform subtraction. Subtract as usual.

$7 + 1 - 3 = 5_7$

$3 - 1 = 2_7$

$7 + 2 - 6 = 3_7$

$6241_7 - 613_7 = 5325_7$

$6241_7 \rightarrow 2\ 1\ 8\ 5_{10}$

$613_7 \rightarrow -\ 3\ 0\ 4_{10}$

$1\ 8\ 8\ 1_{10}$

7	1881
7	268 – 5
7	38 – 2
7	5 – 3
	0 – 5

$6241_7 - 613_7 = 5325_7$

(c) $372_8 - 77_8$

$$\begin{array}{r} {\scriptstyle 2\ 6\ 8} \\ 3\not{7}2_8 \\ -\ \ 7\ 7_8 \\ \hline 2\ 7\ 3_8 \end{array}$$

Transfer numbers according to base eight. Use 8 to perform subtraction. Subtract as usual.

$8 + 2 - 7 = 3_8$

$8 + 6 - 7 = 7_8$

$372_8 - 77_8 = 273_8$

$372_8 \rightarrow 2\ 5\ 0_{10}$

$77_8 \rightarrow -\ 6\ 3_{10}$

$1\ 8\ 7_{10}$

8	187
8	23 – 3
8	2 – 7
	0 – 2

$372_8 - 77_8 = 273_8$

(d) $1827_9 - 65_9$

$$\begin{array}{r} {\scriptstyle 7\ 9} \\ 1\not{8}2\ 7_9 \\ -\ \ 6\ 5_9 \\ \hline 1\ 7\ 5\ 2_9 \end{array}$$

Transfer numbers according to base nine. Use 9 to perform subtraction. Subtract as usual.

$7 - 5 = 2_9$

$9 + 2 - 6 = 5_9$

$1827_9 - 65_9 = 1752_9$

$1827_9 \rightarrow 1\ 4\ 0\ 2_{10}$

$65_9 \rightarrow -\ 5\ 9_{10}$

$1\ 3\ 4\ 3_{10}$

9	1343
9	149 – 2
9	16 – 5
9	1 – 7
	0 – 1

$1827_9 - 65_9 = 1752_9$

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